

The $\xi(2230)$ Meson and The Pomeron Trajectory

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(October 23, 1999)

Abstract

We examine the possibility that the $\xi(2230)$ meson is a member of the Pomeron trajectory. A method of connecting the $\xi \rightarrow p\bar{p}$ decay width and the pp cross sections through the Pomeron residue function is presented. We have used a relativistic, singularity-free form factor to make the analytic continuation of the residue function between crossed channels. We predict that if the $\xi(2230)$ meson is a Pomeron, then it should have a $\xi \rightarrow p\bar{p}$ decay width of about 2 MeV.

Keywords: Regge poles, Pomeron, mesons.

PACS: 12.39Mk, 12.40.Nn, 12.40.Yx, 12.90.+b

Prior to the advent of the quantum chromodynamic (QCD) theory, the Regge theory was extensively used in describing the strong interaction. In spite of its success, the nature of the Pomeron, which plays a crucial role in explaining the asymptotic behavior of the hadron-hadron interaction, remains unclear. Over the years, precise phenomenological fits to high-energy data have determined that [1] [2] the spin and the mass of the Pomerons satisfy the relation $\alpha(t) = \alpha(0) + \alpha' t$ with $\alpha(0) \simeq 1.08$ and $Re[\alpha'] \simeq 0.2(\text{GeV})^{-2}$. If the Pomeron(\mathcal{P}) trajectory is more than a theoretical device, then it should have member constituencies identifiable with physical states. To date, no such states have been found.

In this report we show that the $\xi(2230)$ meson could be a good candidate for Pomeron. This meson was first observed in the radiative decay of the J/Ψ to $K\bar{K}$ by the MARK III collaboration [3]. Interest in $\xi(2230)$ resurged [4] when the BES collaboration [5] measured the reaction $J/\Psi \rightarrow \gamma\xi$ and determined that this state has a mass of 2230 MeV with a total width $\Gamma_{tot} = 20 \pm 17$ MeV. Its spin is still uncertain [6], being either 2^{++} or 4^{++} , but will be determined in future measurements.

If $J = 2$ is confirmed, then with $J = Re[\alpha(t)] = 2$ at $t = M_\xi^2 = (2.23)^2 = 5(\text{GeV})^2$ the ξ satisfies the above-mentioned spin-mass relation of the Pomeron. In the following we shall go beyond this relation and show what would be the constraint on the $\xi \rightarrow p\bar{p}$ decay width, $\Gamma_{\xi \rightarrow p\bar{p}}$, when ξ is on the \mathcal{P} -trajectory. In particular, we show how we can relate it to the residue function of the Pomeron. We then use this relation and the measured pp cross sections to predict the width. In our analysis, we have used for the first time a relativistic and singularity-free form factor. We emphasize that our analysis does not rely on any assumption about the subhadronic content of the ξ meson. Hence, it is model-independent of the latter. The significance of our result will be elucidated.

Let us examine the pp elastic scattering (denoted $12 \rightarrow 34$) mediated by the exchange of the $\xi(2230)$ meson. The corresponding t -channel process is, therefore, the $p\bar{p}$ scattering (denoted $1\bar{3} \rightarrow \bar{2}4$) via the formation and decay of the ξ . Let $m(0.94 \text{ GeV})$, $M_\xi(2.23 \text{ GeV})$, and J_ξ be, respectively, the mass of the $p(\bar{p})$, the mass of the ξ , and the spin of the ξ . The t -channel helicity Feynman amplitude can be written as

$$M_{\lambda_2\lambda_4;\lambda_1\lambda_3}(\bar{s}, \bar{t}) = 4m^2 C_I [4\pi(2J_\xi + 1)] < \lambda_2\lambda_4 | \mathcal{M}^{J_\xi}(\bar{s}) | \lambda_1\lambda_3 > d_{\lambda\lambda'}^{J_\xi}(z_t) \quad (1)$$

where $\lambda \equiv \lambda_1 - \lambda_3$, $\lambda' \equiv \lambda_2 - \lambda_4$, and $C_I = 1/2$ is the isospin factor. For clarity, we denote the square of the total c.m. energy and the momentum transfer in the s -channel by s and t , respectively, and those in the t -channel by $\bar{s} = (p_1 + p_3)^2$ and $\bar{t} = (p_1 - p_2)^2$. In the c.m. of the $p\bar{p}$, $\bar{s} = 4(m^2 + k_t^2)$, $\bar{t} = -2k_t^2(1 - \cos\theta_t) \equiv -2k_t^2(1 - z_t)$. Because $p_3 = -p_1$, $-p_2 = p_1$, we have $\bar{s} = t$ and $\bar{t} = s$. In Eq.(1)

$$< \lambda_2\lambda_4 | \mathcal{M}^{J_\xi}(\bar{s}) | \lambda_1\lambda_3 > = \frac{G_{\lambda'} H_{\lambda_2\lambda_4; J_\xi}(\bar{s}) G_{\lambda} H_{J_\xi; \lambda_1\lambda_3}(\bar{s})}{\bar{s} - M_\xi^2 + iM_\xi \Gamma_{tot}}, \quad (2)$$

The H denotes the form factor for the $\xi p\bar{p}$ vertex in the helicity basis and G the coupling constant.

The Regge-pole amplitude due to the \mathcal{P} -trajectory can be written as [2]

$$A_{\lambda_2\lambda_4;\lambda_1\lambda_3}(\bar{s}, \bar{t}) = C_I \frac{-4\pi^2(2\alpha + 1)\beta_{\lambda\lambda'}(t)(-1)^{\alpha+\lambda}\frac{1}{2}[1 + (-1)^\alpha]}{\sin\pi(\alpha + \lambda')} d_{\lambda\lambda'}^\alpha(z_t) \equiv \mathcal{A}_{\lambda\lambda'}^t, \quad (3)$$

where $\beta_{\lambda\lambda'}(t)$ stands for $\beta_{\lambda_2\lambda_4;\lambda_1\lambda_3}(\bar{s} \equiv t)$. The contribution of the ξ meson to the \mathcal{P} -trajectory is obtained by calculating their overlap as follows:

$$\frac{1}{2} \int_{-1}^{+1} M_{\lambda_2\lambda_4;\lambda_1\lambda_3}(\bar{s}, \bar{t}) d_{\lambda\lambda'}^{J_\xi}(z_t) dz_t = \lim_{\alpha \rightarrow J_\xi} \frac{1}{2} \int_{-1}^{+1} A_{\lambda_2\lambda_4;\lambda_1\lambda_3}(\bar{s}, \bar{t}) d_{\lambda\lambda'}^{J_\xi}(z_t) dz_t. \quad (4)$$

By using $\int_{-1}^{+1} d_{\lambda\lambda'}^{J_\xi}(z_t) d_{\lambda\lambda'}^{J_\xi}(z_t) dz_t = 2/(2J_\xi + 1)$ and the following relation, which we have derived,

$$\frac{1}{2} \int_{-1}^{+1} d_{\lambda\lambda'}^\alpha(z_t) d_{\lambda\lambda'}^{J_\xi}(z_t) dz_t = \frac{\sin\pi\alpha}{\pi(\alpha - J_\xi)(\alpha + J_\xi + 1)} \mathcal{F} \quad (5)$$

with

$$\mathcal{F} \equiv (-1)^{J_\xi} \left(\frac{(\alpha - \lambda)!(J_\xi + \lambda)!}{(\alpha + \lambda)!(J_\xi - \lambda)!} \right)^{\frac{1}{2}}, \quad (\lambda = 0, \pm 1), \quad (6)$$

we obtain

$$\begin{aligned} & \frac{16m^2\pi G^2 H_{\lambda_2\lambda_4; J_\xi}(t) H_{J_\xi; \lambda_1\lambda_3}(t)}{t - M_\xi^2 + iM_\xi \Gamma_{tot}} \\ &= \left[\frac{-4\pi(2\alpha + 1)\beta_{\lambda\lambda'}(t)}{(\alpha - J_\xi)(\alpha + J_\xi + 1)} \frac{(-1)^{\alpha+\lambda}\frac{1}{2}[1 + (-1)^\alpha]\sin\pi\alpha}{\sin\pi(\alpha + \lambda')} \mathcal{F} \right]_{\alpha \rightarrow J_\xi} \\ &= \frac{-4\pi\beta_{\lambda\lambda'}(t_r)/\alpha'_R}{t - t_r + i\alpha_I(t_r)/\alpha'_R}. \end{aligned} \quad (7)$$

Hence, for even J_ξ we have the identifications $t_r = M_\xi^2$, $\alpha_I(t_r)/\alpha'_R = M_\xi \Gamma_{tot}$, and

$$\beta_{\lambda\lambda'}(t_r) = -\alpha'_R 4m^2 G_{\lambda'} H_{\lambda_2\lambda_4;J_\xi}(t_r) G_\lambda H_{J_\xi;\lambda_1\lambda_3}(t_r) . \quad (8)$$

In obtaining Eq.(7) we have defined $\alpha_R \equiv \text{Re}(\alpha)$, $\alpha_I \equiv \text{Im}(\alpha)$, and have used the Taylor expansion $\alpha(t) = \alpha_R(t_r) + (\alpha'_R + i\alpha'_I(t_r))(t - t_r) + \alpha_I(t_r) + \dots$, with $\alpha_R(t_r) \equiv J_\xi$, $\alpha_I \ll \alpha_R$, and $\alpha'_I \ll \alpha'_R$.

The diagonal matrix element of Eq.(8) is directly related to the $\xi \rightarrow p\bar{p}$ decay width. Using the method of [7], we obtain

$$\begin{aligned} \frac{1}{2}\Gamma_{\xi \rightarrow p\bar{p}} &= C_I \frac{m^2}{M_\xi^2} \frac{|\mathbf{q}_r|}{16\pi^2} \sum_{\lambda_p\lambda_{\bar{p}}} |G_\lambda H_{\lambda_p\lambda_{\bar{p}}}(\mathbf{q}_r^2)|^2 \\ &= C_I \frac{|\mathbf{q}_r|}{64\pi^2\alpha'_R M_\xi^2} \sum_{\lambda} \beta_{\lambda\lambda}(t_r) , \end{aligned} \quad (9)$$

where $\mathbf{q}_r^2 = t_r/4 - m^2 = 0.36 \text{ (GeV)}^2$ and $\lambda = \lambda_p - \lambda_{\bar{p}}$. The \mathbf{q}_r is the value of the c.m. momentum \mathbf{q} of the $p\bar{p}$ system at $\bar{s} = M_\xi^2 \equiv t_r$. On the other hand, the $\beta_{\lambda\lambda'}(t)$ at $t \leq 0$ is related to the pp cross sections. The spin-averaged total pp cross section is given by

$$\sigma_{tot}^{pp} = \left(\frac{1}{4}\right) \left(\frac{4\pi}{k_s}\right) \frac{1}{8\pi k_s} \sum_{\lambda_3\lambda_4;\lambda_1\lambda_2} \text{Im}[\mathcal{A}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^s(s, t)]|_{t=0} , \quad (10)$$

where k_s and \mathcal{A}^s are the s -channel c.m. momentum and amplitudes, and $k_s^2 \simeq s/4$. Furthermore, $\sum_{\lambda_3\lambda_4;\lambda_1\lambda_2} \text{Im}[\mathcal{A}_{\lambda_3\lambda_4;\lambda_1\lambda_2}^s] = \sum_{\lambda_2\lambda_4;\lambda_1\lambda_3} \text{Im}[\mathcal{A}_{\lambda_2\lambda_4;\lambda_1\lambda_3}^t]$ due to crossing symmetry. There are sixteen helicity amplitudes for $p\bar{p}$ scattering. However, only five amplitudes are independent as a result of the parity, total spin conservation, and time-reversal invariance. We denote them as \mathcal{A}_i^t . The indices $i = 1, \dots, 5$ correspond successively to $(\lambda_2\lambda_4; \lambda_1\lambda_3) = (++; ++)$, $(++; --)$, $(+-; +-)$, $(+-; -+)$, $(++; +-)$ with \pm denoting, respectively, the helicities $\pm\frac{1}{2}$. In the limit $s \rightarrow \infty$ and, hence, $z_t \rightarrow \infty$ [2]

$$d_{\lambda\lambda'}^\alpha \sim \frac{(-1)^{(\lambda-\lambda')/2} (2\alpha)! (z_t/2)^\rho}{\sqrt{(\alpha+M)! (\alpha-M+|\lambda-\lambda'|)! (\alpha-M)! (\alpha+M-|\lambda-\lambda'|)!}} \quad (11)$$

and

$$\left(\frac{z_t}{2}\right)^\rho \sim (-1)^\rho \left(\frac{s}{4m^2 - t}\right)^\rho \quad (12)$$

where $\rho \equiv \alpha - M + |\lambda - \lambda'|/2 + |\lambda + \lambda'|/2$ and $M \equiv \max(|\lambda|, |\lambda'|)$. Upon introducing Eqs.(11) and (12) into Eq.(3), one sees that $\mathcal{A}_1^t = \mathcal{A}_2^t$ and $\mathcal{A}_3^t = -\mathcal{A}_4^t$. Consequently,

$$\sigma_{tot}^{pp} = \frac{1}{2s} \text{Im}[4\mathcal{A}_1^t + 8\mathcal{A}_5^t] |_{t=0} . \quad (13)$$

The pp elastic differential cross section is given by

$$\frac{d\sigma^{pp}}{dt} = \frac{1}{16\pi s^2} (4|\mathcal{A}_1^t|^2 + 4|\mathcal{A}_3^t|^2 + 8|\mathcal{A}_5^t|^2) . \quad (14)$$

In the above equations, $\mathcal{A}_1^t, \mathcal{A}_3^t, \mathcal{A}_5^t$ depend, respectively, on $\beta_{00}(t), \beta_{11}(t), \beta_{10}(t)$.

The $\beta(t)$ can be calculated from $\beta(t_r)$ with the use of a model that we specify below. First, we note that in deriving Eq.(8) one only requires $\lim_{t \rightarrow t_r} \alpha_R(t) = J_\xi$. From the mathematical point of view, the position of t at which the limit is taken can be anywhere along the Regge/Pomeron trajectory. Hence, we have the functional equality

$$\beta_{\lambda\lambda'}(t) = -\alpha'_R 4m^2 G_{\lambda'} H_{\lambda_2\lambda_4;J_\xi}(t) G_\lambda H_{J_\xi;\lambda_1\lambda_3}(t) \quad (15)$$

along this trajectory. In fact, the theory of analytic functions implies that the equality exists in a region where both β and H are analytic functions. In general the functional form of $H(t)$ can also depend on t . However, because there are no bound states and other resonances at $t < M_\xi^2$, we can assume that the functional form of $H(t)$ is the same in the entire region $t \leq M_\xi^2$. From Eqs.(8) and (15) we have

$$\beta_{\lambda\lambda'}(t) = \beta_{\lambda\lambda'}(t_r) \frac{H_{-\lambda_4,\lambda_4;J_\xi}(t) H_{J_\xi;\lambda_1,-\lambda_1}(t)}{H_{-\lambda_4,\lambda_4;J_\xi}(t_r) H_{J_\xi;\lambda_1,-\lambda_1}(t_r)} . \quad (16)$$

Once the form factor H is known, the $\beta_{\lambda\lambda'}$ can be calculated from their values at t_r which, by Eq.(9), are related to the decay width. It is advantageous to model the form factor in the LS -basis because in this latter basis the form factor has a well-known q^L threshold behavior that can be explicitly incorporated into the model. Because the helicity basis is related to the LS -basis by a unitary transformation [8], one has $\sum_\lambda |G_\lambda H_{\lambda_p,\lambda_{\bar{p}};J_\xi}|^2 = \sum_{LS} |g_{LS} F_{LS}|^2$ in Eq.(9). Here F_{LS} and g_{LS} denote, respectively, the form factor and the dimensionless coupling constant in the LS -basis. For $p\bar{p}$ system with $J^P = 2^+$, the parity conservation leads to $L = 1, 3$ and $S = 1$ only. We will henceforth omit the subscript S .

In the literature, the most commonly used hadronic form factor is either of an exponential form or of a multipole form. However, these form factors are unsuitable to analyses involving channel-crossing. For example, the exponential form factor $\exp(t/\Lambda^2)$ is analytic in the s -channel where $t \leq 0$ but diverges in the t -channel where $t > 0$. The multipole form factor $[\Lambda^2/(\Lambda^2-t)]^n$ ($n = 1, 2, \dots$) has no pole on the real axis of t in the s -channel (where $t \leq 0$) but will have it in the t -channel (where $t > 0$). Conversely, if we use $\exp(-t/\Lambda^2)$ or $[\Lambda^2/(\Lambda^2+t)]^n$, then the situation will be reversed. We propose the following form factor which is singularity free:

$$|F_L(t)|^2 = \left(\frac{t/4 - m^2}{\mathbf{q}^2} \right)^L \left(\frac{e^{t_r/\lambda_t^2}}{R(-x_t) + e^{t/\lambda_t^2}} \right)^2 \left(\frac{1 + e^{-t_r/\lambda_s^2}}{R(x_s) + e^{-t/\lambda_s^2}} \right)^2 \quad (17)$$

where $(t/4 - m^2)^L \equiv f_L = \mathbf{q}^{2L}$ reflects the q^L -dependence of the F_L . Because $F_L(t_r) \equiv 1$ we have from Eq.(9) and from the relation between H and F that $\Gamma_{\xi \rightarrow p\bar{p}} \propto g_1^2 + g_3^2$.

In Eq.(17), $x_s \equiv (t - 2m^2)/\lambda_s^2$ and $x_t \equiv (t - 2m^2)/\lambda_t^2$. The function R is analytic and is defined by

$$R(x) = \frac{1}{2}(1 + \tanh(ax)) = \frac{e^{ax}}{e^{ax} + e^{-ax}}. \quad (18)$$

It rapidly changes from 0 to 1 when x changes from < 0 to > 0 , with a controlling the transition speed at $x = 0$. With $a > 10$, R will be very close to a step function but does not have the discontinuity of the latter. Consequently, the form factor is a continuous function of t . In the s -channel ($t \leq 0$), $|F(t)|^2 \propto f_L [1 + \exp(t/\lambda_t^2)]^{-2} [\exp(-t/\lambda_s^2)]^{-2} \sim t^L \exp(2t/\lambda_s^2)$, which goes to 0 as $t \rightarrow -\infty$. Hence, λ_s controls the form factor. Since $F(t)$ does not have the physical-channel energy s as an explicit variable, it does not diverge with s . When t reaches the t -channel physical domain ($t > 4m^2$), $|F(t)|^2 \propto f_L [\exp(t/\lambda_t^2)]^{-2} [1 + \exp(-t/\lambda_s^2)]^{-2} \sim t^L \exp(-2t/\lambda_t^2) \rightarrow 0$ when $t \rightarrow \infty$, exhibiting the correct energy behavior in the t -channel. Here in the t -channel the λ_t controls the form factor. The above well-behaved t -dependence of $F(t)$ makes the latter a good tool for continuing β between the direct and crossed channels.

Eqs.(9) and (13) to (16) allow us to predict the decay width $\Gamma_{\xi \rightarrow p\bar{p}}$ from the measured pp total and elastic cross sections, and vice versa. We have determined the form factor

parameters from the experimental pp total and elastic cross sections data [1] [6] at $\sqrt{s} = 53$ and 62 GeV. As anticipated, λ_s is mainly determined by the diffraction peak of the $d\sigma/dt$.

The result of our fit is given in Table I where the wider parameter ranges at 62 GeV are due to the larger experimental error bars. Table I indicates that if ξ lies on the \mathcal{P} -trajectory, then it has a $\Gamma_{\xi \rightarrow p\bar{p}}$ between 1.5 and 2 MeV. This decay width, in addition to spin, can be used to check whether ξ is a Pomeron. We note that the BES collaboration [5] cited nearly equal branching ratios for the four observed decays modes. Using these equal branching ratios and the above $\Gamma_{\xi \rightarrow p\bar{p}}$ we conclude that the Γ_{tot} for the ξ is at least 8 MeV. While this lower bound is compatible with the published data, it is, however, necessary to ascertain in future experiments that no other important decay channels than those observed in ref. [5] are left out.

We recall that the theoretical modeling of the Pomeron started in the perturbative QCD sector [9]– [11]. Various gluon-exchange models were proposed. However, all these models gave an $\alpha(0)$ much greater than the phenomenological value of 1.08. Recently, there is a growing interest in a possible connection between the Pomeron and the glueball [12]. We emphasize that at this time there is no convincing proof that Pomeron is a glueball. As to the $\xi(2230)$ meson itself, it can either be a tensor glueball or a $q\bar{q}$ -glue mixture [13]– [17]. Frank Close [15] has shown that if ξ is a tensor glueball then its total width Γ_{tot} would be of the order of 25 MeV; but no discussion was made on the $\Gamma_{\xi \rightarrow p\bar{p}}$. Although the Γ_{tot} of the BES result [5] falls within this limit, that measurement needs to be improved [4]. We believe that high-statistics data on many decay modes of all the tensor states in this mass region are needed to pin down the gluonic content of the ξ . In this work, we do not study the microscopic composition of the ξ . Instead, we investigate the conditions that ξ could be a Pomeron. If the Pomeron status of the ξ is established, then the subhadronic structure of the ξ will be directly relevant to that of the Pomeron. In this respect, our study will help clarify the Pomeron-glue connection.

In summary, we have derived the relation between the residue function of the Pomeron trajectory and decay widths of its member states. Measurements of the spin and the $p\bar{p}$ -decay

width of the ξ are important for determining if the $\xi(2230)$ meson could be the long-sought Pomeron.

We thank Drs. J.C. Peng, L.S. Kisslinger, P.R. Page, and L. Burakovsky for helpful discussions.

TABLES

TABLE I. The $\Gamma_{\xi \rightarrow p\bar{p}}$ predicted from the measured pp cross sections

\sqrt{s} (GeV)	λ_s (GeV)	λ_t (GeV)	g_1	g_3	$\Gamma_{\xi \rightarrow p\bar{p}}$ (MeV)
53	0.65–0.67	3.43–3.44	1.13–1.14	0.295–0.299	1.85–1.86
62	0.66–0.68	3.25–4.07	1.07–1.19	0.277–0.360	1.65–2.08

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